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International Journal of Heat and Mass Transfer 46 (2003) 2329–2339

International Journal of HEAT and MASS TRANSFER

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Influence of viscosity variations on the forced convection flow through two types of heterogeneous porous media with isoflux boundary condition

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Received 21 March 2002; received in revised form 6 December 2002

Abstract

A theoretical analysis of fully developed forced convection flow through a heterogeneous porous medium with isoflux boundary condition is carried out by invoking a varying viscosity model to determine its effect on the resulting heat and fluid flow characteristics. Two different types of permeability and thermal conductivity variations, in the transverse direction of the channel, are accounted for in the analysis, viz., continuous weak and step-wise variations. Analytical solutions are obtained and graphical illustrations are presented to reveal the influence of thermal conductivity, permeability and viscosity variations. The results indicate that the viscosity variations have significant effect on the resulting heat transfer characteristics interpreted in terms of the Nusselt number. Moreover, the results reveal that the heat transfer rate decreases with the increase of permeability ratio for strong viscosity variations, which cannot be captured if a constant viscosity model is assumed.

1. Introduction

Heat and fluid flow through porous materials occur in a large number of industrial applications ranging from simulation of petroleum reservoirs, packed bed catalytic reactors, porous insulation and heat pipe wicking structures, and reviews are well documented [1,2]. Recent applications of porous media flows include the flow of liquids in biological and physiological processes, pebble-bed nuclear reactors and cooling of turbine blades in the hot portion of a turbo-expander. For simple problems, the theory of homogeneous porous medium may be sufficient, but for complicated geometry a thorough understanding of the fluid flows in both homogeneous and heterogeneous porous media becomes essential. This motivated a few to discuss heat and fluid flow in heterogeneous porous media. Detailed reviews of heat and fluid flow through anisotropic porous layers are found in Storesletten [3], Vasseur and Robillard [4], Vafai [5]. Vadasz [6] is one of the first to study the fluid flow through heterogeneous porous media in a rotating square channel, with reference to flows in packed-bed mechanically agitated vessels that are widely used in food processing and chemical engineering industries. His investigations reveal that for a heterogeneous porous medium with spatial dependent permeability, the flow is no longer irrotational and therefore the effects of rotation become most important, unlike the flows through homogeneous porous media. Continued work resulted in Vadasz and Havstad [7] and Havstad and Vadasz [8].

More recently, evaporation heat transfer in bi-porous media was theoretically analyzed by Wang and Catton [9] with reference to thermal management in electronics chips, laser diode arrays and high power optics, where it is necessary to cool the heat sources whose heat flux exceeds 100 W/cm^2. Their results revealed that the fluid flow in capillaries covering a bi-porous medium was capable of removing higher order heat flux as mentioned above. Generally, the development of micro-scale and nano-scale technology has led to the demand of high performance heat transfer devices in order to remove

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doi:10.1016/S0017-9310(02)00536-7

Nomenclature

c_p	specific heat at constant pressure
Da	Darcy number
G	applied pressure gradient
H	half channel width
k	thermal conductivity
$\overline{k}$	mean value of k
$\tilde{k}$	$k/\overline{k}$
K	permeability
$\overline{K}$	mean value of K
$\tilde{K}$	$K/\overline{K}$
Nu	Nusselt number
p'	dimensional pressure
Pe	Peclet number
q''	wall heat flux at the outer boundary
u'	dimensional velocity
$\overline{u}$	dimensionless velocity
U'	dimensional mean velocity
T'	dimensional temperature
T'_{m}	dimensional bulk mean temperature
T'_{w}	dimensional wall temperature
T_∞	reference temperature
$\overline{T}$	dimensionless temperature
x', y'	dimensional coordinates
x, y	dimensionless coordinates

Greek symbols

α	temperature coefficient of viscosity defined in Eq. (A.1)
β	viscosity variation parameter defined in Eq. (A.5)
γ	a constant defined in Eq. (A.3)
ε_k, ε_K	coefficients defined in Eq. (12a,b)
ρ	density
μ	fluid viscosity
μ_0	reference fluid viscosity
ξ	dimensionless coordinate defined in Fig. 1
ζ	a constant defined in Eq. (5b)

Subscripts

1, 2	referring to regions 1 and 2 in Fig. 1, respectively

high density thermal energy and avoid thermal damages due to heat accumulations and this also encouraged fundamental studies on heat and fluid flow in heterogeneous porous media. Rees and Postelnicu [10] discussed the onset of convection in an inclined porous layer, assumed to be anisotropic with respect to both permeability and diffusivity. They determined not only how much the critical Rayleigh number varies when the layer is inclined but also the wave number and the angle that the fluid roll makes with the direction at which the layer is inclined.

Nield and Kuznetsov [11] studied the global heterogeneity effects on the fully developed flow between parallel plates occupied by a heterogeneous porous medium in the presence of isoflux and isotemperature boundaries. They considered two types of permeability and thermal conductivity, namely, weak continuous and step-wise variations. Their investigations demonstrated that the effect of permeability variation is that an above-average permeability near the walls leads to an increase in the Nusselt number. Furthermore, the effect of thermal conductivity variation was found to be more complex with two opposing effects and the Nusselt number was found to be not always a monotonic function of conductivity variation. Thus the concept of heterogeneity in the porous medium may be used to replace the homogeneous porous medium employed in optimizing the critical heat flux that allows re-wetting in micro-heat pipes (Peterson [12]). The extent to which the heterogeneity effects may be exploited in such an application is still not known completely.

Earlier studies by Ramirez and Saez [13] and Seddeek [14], discussed the influence of temperature dependent viscosity variations on the resulting heat and fluid flows in a homogeneous porous medium. All these investigations concluded that the effect of variation in viscosity should be taken into account, even for relatively low temperature gradients, to match the experimental observations. The importance of inclusion of viscosity variation in thermal analysis, especially in porous media, is such that for example, estimation of the effective thermal conductivity in water results in variations of the order of 50%, when their respective viscosity is assumed to be a constant and when compared with that of the varying viscosity model [13]. Also, as the Peclet number is defined through thermal conductivity, its effect on the Nusselt number is strong especially for values of the viscosity variation parameter $\delta(=\alpha T_\infty$, as in [13], where α is the temperature coefficient of viscosity and T_∞ is the reference temperature) around -1. For instance, the calculated value of the Nusselt number Nu for $\delta = 0$ (constant viscosity model) and the Peclet number $Pe = 100$ is 6.5, and for $\delta = -1$ (varying viscosity model) and $Pe = 100$, the Nu is found to be around 2 [13].

This made us realize the importance of viscosity variations not only in a homogeneous porous medium, but also in a heterogeneous medium; and thus motivated the present study, in which the work of Nield and

Kuznetsov is extended to include temperature dependent properties, as presented below.

2. The Model

The permeability and thermal conductivity are allowed to vary non-uniformly in space and therefore are defined respectively as

$$\widetilde{K} = \frac{K}{\overline{\overline{K}}}, \quad \tilde{k} = \frac{k}{\bar{\bar{k}}}, \tag{1a, b}$$

where an over bar denotes a mean value taken over the volume occupied by the porous medium. It is assumed that for this steady fully developed situation, we have an unidirectional flow with velocity u' in the x' as shown in Fig. 1. Due to axis-symmetric assumption in the heat and fluid flow behaviors, only upper half of the channel is depicted in Fig. 1 and the subsequent analysis is also carried out only in this region without loss of generality. We further assume that K and k are functions of y' only. The Darcy equation for the pressure gradient is then written as

$$G = \frac{\mu}{K}u', \tag{2}$$

where $G = -\mathrm{d}p/\mathrm{d}x$. The use of this model here is not only to simplify the analysis but also to compare with that of the earlier model by Nield and Kuznetsov [11], wherein they restricted to the Dupuit–Darcy–Forcheimer model and discussed cases for which it reduced to the Darcy model. From the investigations of Nield and Kuznetsov, the distribution of temperature in space (across the channel) is found to be almost in a linear fashion (i.e., decreases with increase of y'), especially for values of K_2/K_1 and k_2/k_1 around 1. Hence the linear dependence of viscosity on temperature, assumed for most of the common fluids, is approximated as a linear dependence on y (dimensionless form of y') as,

$$\mu = \mu_0[(1-\beta) + \beta y], \tag{3}$$

where μ_0 is the reference fluid viscosity computed at the channel wall temperature ($y' = H$) and the constant β is assumed to be small when compared to unity (for details see Appendix A).

Substituting Eq. (3) in to Eq. (2), and using the variables

$$x = \frac{x'}{H}, \quad y = \frac{y'}{H}, \quad u = \frac{\mu_0 u'}{GH^2},$$

the dimensionless form of Eq. (2), together with the viscosity variation, can be written as

$$u = \frac{\widetilde{K}}{(1-\beta)}Da\left(1 + \frac{\beta}{(1-\beta)}y\right)^{-1}, \tag{4}$$

where the Darcy number is defined by $Da = \overline{K}/H^2$.

By defining the mean velocity and bulk mean temperature (a velocity-weighted average temperature over the channel cross section) respectively as [15,16]

$$U' = \frac{1}{H}\int_0^H u'\,\mathrm{d}y', \quad T'_{\mathrm{m}} = \frac{1}{HU'}\int_0^H u'T'\,\mathrm{d}y',$$

and further defining dimensionless variables

$$\bar{u} = \frac{u'}{U'} \quad \overline{T} = \frac{T' - T'_{\mathrm{w}}}{T'_{\mathrm{m}} - T'_{\mathrm{w}}},$$

it can be shown that

$$\bar{u} = \widetilde{K}\left\{\left[1 - \left(\frac{\beta}{1-\beta}\right)y\right] + \left(\frac{\beta}{2(1-\beta)}\right)\right\}. \tag{5a}$$

i.e.,

$$\bar{u} = \widetilde{K}\left\{1 - \zeta\left(y - \frac{1}{2}\right)\right\}, \tag{5b}$$

where $\zeta = [\beta/(1-\beta)]$ is a constant.

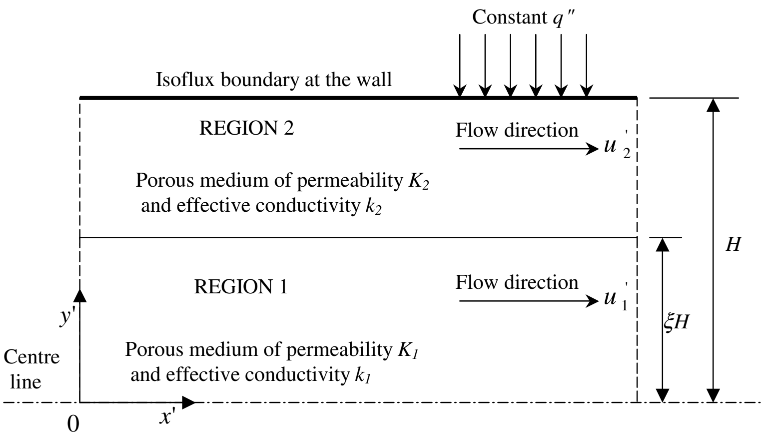

Fig. 1. Upper half of parallel plate channel (for step-wise variation case).

The heat transfer rate calculated in terms of Nusselt number can be defined as

$$Nu = \frac{2Hq''}{\bar{k}(T'_w - T'_m)}. \tag{6}$$

The thermal energy equation is

$$u'\frac{\partial T'}{\partial x'} = \frac{1}{\rho c_p}\frac{\partial}{\partial y'}\left(k\frac{\partial T'}{\partial y'}\right), \tag{7a}$$

i.e.,

$$u'\frac{\partial T'}{\partial x'} = \frac{1}{\rho c_p}\frac{\partial k}{\partial y'}\left(\frac{\partial T'}{\partial y'}\right) + \frac{k}{\rho c_p}\left(\frac{\partial^2 T'}{\partial y'^2}\right), \tag{7b}$$

since k is considered to vary across the channel height. The exclusion of the axial conduction term in the above thermal energy equation has been justified for cases of flow in uniform wall heat flux or uniform wall temperature, when the velocity profile is similar to that of slug flow, which is presently the case for flow in porous medium [17].

Now the gradient of k in Eq. (7b) is zero when it is assumed to vary in a step-wise manner in space (see Section 2.2). In the case of continuous variation (Section 2.1), the first term, involving $(\partial k/\mathrm{d}y')$, on the right hand side of Eq. (7b) is neglected in the present analysis by assuming that its value is small when compared with the second term.

Applying the first law of thermodynamics to a thermally fully developed flow with isoflux boundary condition, we have the results [15,16]

$$\frac{\partial T'}{\partial x'} = \frac{\partial T'_m}{\partial x'} = \frac{\partial T'_w}{\partial x'} = \frac{q''}{\rho c_p H U} = \text{constant}, \tag{8}$$

where it is noted that the dimensional temperature gradient in the axial direction is non-zero.

Thus the thermal energy equation (7b) becomes, with the aid of Eqs. (5), (6) and (8), and in dimensionless form,

$$\frac{\mathrm{d}^2\overline{T}}{\mathrm{d}y^2} = -\frac{1}{2\tilde{k}}Nu\bar{u} \tag{9}$$

and for the Darcy flow in the presence of viscosity variation, it is

$$\frac{\mathrm{d}^2\overline{T}}{\mathrm{d}y^2} = -\frac{1}{2\tilde{k}}Nu\widetilde{K}\left(1+\zeta\left(y-\frac{1}{2}\right)\right)^{-1}. \tag{10}$$

The first boundary condition for Eq. (10) is, by symmetry,

$$\left.\frac{\mathrm{d}\overline{T}}{\mathrm{d}y}\right|_{y=0} = 0. \tag{11a}$$

The second boundary condition is the isoflux condition at $y' = H$. This is a constant $q''\left(= \rho c_p H U'(\mathrm{d}T'_w/\mathrm{d}x')\right)$, which can alternatively be expressed as

$$\left.\overline{T}\right|_{y=1} = 0, \tag{11b}$$

because (11b) implies that at the wall, $T' = T'_w$, where T'_w varies linearly for the case of isoflux condition in thermally developed flow, as described by Eq. (8).

2.1. Continuous weak variation case

We first consider the continuous weak variation case where the permeability and thermal conductivity distributions are varying slightly away from their constant values and are

$$K = \overline{K}\left\{1+\varepsilon_K\left(\frac{y'}{H}-\frac{1}{2}\right)\right\}, \quad k = \bar{k}\left\{1+\varepsilon_k\left(\frac{y'}{H}-\frac{1}{2}\right)\right\}, \tag{12a, b}$$

where the coefficients ε_K and ε_k are each assumed to be small compared with unity. Then dimensionless form of Eqs. (12a,b) are

$$\widetilde{K} = 1+\varepsilon_K\left(y-\frac{1}{2}\right), \quad \tilde{k} = 1+\varepsilon_k\left(y-\frac{1}{2}\right). \tag{13a, b}$$

Using Eqs. (13a,b) in Eq. (10), and approximating to first order in small quantities, we obtain

$$\frac{\mathrm{d}^2\overline{T}}{\mathrm{d}y^2} = -\frac{Nu}{2}\left\{1+(\varepsilon_K-\varepsilon_k-\zeta)\left(y-\frac{1}{2}\right)\right\}. \tag{14}$$

The solution of Eq. (14) subject to the boundary conditions in Eqs. (11a,b) is

$$\overline{T}(y) = -\frac{Nu}{24}\left\{6(y^2-1)+(\varepsilon_K-\varepsilon_k-\zeta)(2y^3-3y^2+1)\right\}. \tag{15}$$

The determining compatibility condition is

$$\int_0^1 \bar{u}\overline{T}\,\mathrm{d}y = 1. \tag{16}$$

Finally, substitution of the expressions Eqs. (5) and (15) into (16) leads to the first order result

$$Nu = 6\left(1+\frac{1}{4}\varepsilon_K-\frac{1}{8}\varepsilon_k-\frac{\zeta}{4}\right). \tag{17}$$

This reduces to the result of Nield and Kuznetsov [11] on substituting $\zeta = 0$. Further more, for slug flow in an isoflux non-porous channel, it is well-known that $Nu = 6$ $(\varepsilon_K = \varepsilon_k = 0)$, serving as a check to our result.

2.2. Step-wise variation case

For the step-wise variation case, it is assumed that, referring to Fig. 1,

$$K = K_1 \quad \text{and} \quad k = k_1 \quad \text{for } 0 < y' < \xi H, \tag{18a, b}$$

$$K = K_2 \quad \text{and} \quad k = k_2 \quad \text{for } \xi H < y' < H, \tag{19a, b}$$

The mean values are given by

$$\overline{K} = \xi K_1 + (1-\xi)K_2, \qquad \overline{k} = \xi k_1 + (1-\xi)k_2. \tag{20a, b}$$

We write

$$\widetilde{K}_i = \frac{K_i}{\overline{K}} \quad \text{and} \quad \tilde{k}_i = \frac{k_i}{\bar{k}}, \quad \text{for } i = 1, 2.$$

The velocity distributions are then, from Eq. (5b),

$$\bar{u}_1 = \widetilde{K}_1\left(1-\zeta\left(y-\frac{1}{2}\right)\right), \quad \text{for } 0 < y < \xi, \qquad \bar{u}_2 = \widetilde{K}_2\left(1-\zeta\left(y-\frac{1}{2}\right)\right), \quad \text{for } \xi < y < 1. \tag{21a, b}$$

We have now to solve the differential equation pair

$$\frac{d^2\overline{T}_1}{dy^2} = -\frac{1}{2\tilde{k}_1}Nu\widetilde{K}_1\left(1-\zeta\left(y-\frac{1}{2}\right)\right), \quad \text{for } 0 < y < \xi,$$
$$\frac{d^2\overline{T}_2}{dy^2} = -\frac{1}{2\tilde{k}_2}Nu\widetilde{K}_2\left(1-\zeta\left(y-\frac{1}{2}\right)\right), \quad \text{for } \xi < y < 1, \tag{22a, b}$$

subject to boundary conditions

$$\left.\frac{d\overline{T}_1}{dy}\right|_{y=0} = 0, \quad \overline{T}_2\big|_{y=1} = 0, \tag{23a, b}$$

as well as the matching conditions at the interface for both temperature and heat flux:

$$\overline{T}_1(\xi) = \overline{T}_2(\xi), \quad -\tilde{k}_1\frac{\partial\overline{T}_1}{\partial y}(\xi) = -\tilde{k}_2\frac{\partial\overline{T}_2}{\partial y}(\xi). \tag{24a, b}$$

It can be shown that the solution is

$$\overline{T}_1(y) = \frac{Nu}{4}\left\{\frac{\widetilde{K}_1}{\tilde{k}_1}\left[\frac{\zeta}{3}(y^3-\xi^3)+\xi^2-y^2\right] + \frac{\widetilde{K}_2}{\tilde{k}_2}\left[-2\xi+\xi^2+1+\frac{\zeta}{6}(2y^3+9\xi^2-14\xi-6\xi^3+4)\right] + \frac{\widetilde{K}_1}{\tilde{k}_2}\left[2\xi-2\xi^2+\zeta(\xi^3-\xi^2)\right]\right\}$$
$$\overline{T}_2(y) = \frac{Nu}{4}\left\{\frac{\widetilde{K}_2}{\tilde{k}_2}\left[\frac{\zeta}{6}(2y^3-3y^2-9y-6\xi^2y+2\xi y+6\xi^2 - 6\xi+4)+1-2\xi y+2\xi y-y^2\right] + \frac{\widetilde{K}_1}{\tilde{k}_2}\left[2\xi-2\xi y+\zeta(\xi^2y-\xi y-\xi^2+\xi)\right]\right\} \tag{25a, b}$$

Substituting the above into the determining compatibility condition (Eq. (16)) then yields the final Nusselt number expression

$$Nu = 6\Big/\left\{\frac{\widetilde{K}_1^2}{\tilde{k}_1}\left(\xi^3+\frac{3\zeta}{2}\left(-\frac{3\xi^4}{4}+\frac{2}{3}\right)\right) + \frac{\widetilde{K}_1^2}{\tilde{k}_2}\left(3\xi^2(1-\xi)+\frac{3\zeta}{2}(2\xi^2-2\xi^3)\right) + \frac{\widetilde{K}_1\widetilde{K}_2}{\tilde{k}_2}\left[3\xi(1-2\xi+\xi^2)+\frac{3\zeta}{2}\left(-\frac{13}{4}\xi^4 + \frac{7}{2}\xi^3-4\xi^2+\frac{\xi}{2}+\frac{2}{3}\right)\right] + \frac{\widetilde{K}_2^2}{\tilde{k}_2}\left((1-\xi)^3 + \frac{3\zeta}{2}\left(\frac{5}{6}\xi^4-\frac{8}{3}\xi^3+\frac{15}{2}\xi^2-\frac{5}{2}\xi+\frac{1}{12}\right)\right)\right\}. \tag{26}$$

For the homogeneous case and constant viscosity model (i.e. $\xi = 1$, $\zeta = 0$ and $\widetilde{K}_1 = \widetilde{K}_2 = \tilde{k}_1 = \tilde{k}_2 = 1$) this expression again reduces to $Nu = 6$, the slug flow result.

3. Results and discussions

Firstly we re-emphasize that the results are computed for values of K_2/K_1 and k_2/k_1 around one for validity of near linear assumption of temperature distribution in space, so as to assume a linearly varying viscosity model in space. Results are for two different types of thermal conductivity and permeability variations as given in Eqs. (13a,b), (20a,b) and for some fixed values of β, viz., 0, 0.05, 0.2.

The present viscosity model can be checked by computing the viscosity along the channel transverse direction, using Eqs. (3) and (A.2), and by approximating the dimensionless temperature therein (i.e., in Eq. (A.2)) through expressions given in Eqs. (25a,b). Calculated viscosities are graphically shown in Fig. 2, which shows that the present approximation for viscosity, i.e., Eq. (3), may yield satisfactory results, especially for low values of β. In the above calculations, values of β are chosen arbitrarily as 0.05 and 0.2, respectively. The value of γ is obtained by approximating the dimensionless temperature reported in [11] using a linear profile in transverse coordinate and calculating the normal component of temperature gradient (i.e. $\gamma = -d\overline{T}/dy$, see Appendix A). In an actual system, β may be calculated if the bulk mean temperature, channel wall temperature, first order temperature coefficient of viscosity and transverse temperature gradient at any point along the transverse direction are known.

For the continuous weak variation case, the effects of permeability and conductivity variations on temperature and velocity are shown in Fig. 3. Fig. 3(a) depicts the temperature distribution across the channel for a constant viscosity model ($\beta = 0$) and for increasing values of ε_K and ε_k. Increasing values of ε_K or ε_k leads to a slight increase in the temperature distribution across the channel when compared to that of both ε_K and ε_k being

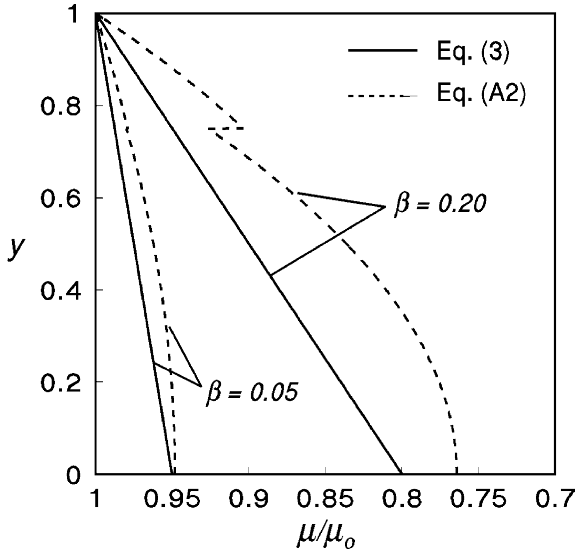

Fig. 2. Comparison of viscosity acquired from two different models.

equal in their magnitudes. A similar trend is also observed in the case of a varying viscosity model ($\beta = 0.2$), except that the temperature is being reduced slightly, Fig. 3(b). This is because of the increase in the local velocities near the walls in the case of $\beta = 0$ when compared to that of non-zero β (see Fig. 3(c) and (d)) and thus resulting in the enhancement of lateral mixing. The velocity distributions for both the constant and variable viscosity model are shown in Fig. 3(c) and (d). For the constant viscosity model case, the velocity is found to decrease in the lower half of the channel (i.e., $0 < y < 0.5$) and increase in the upper half ($0.5 > y > 1$), for increasing values of ε_K. This may be explained through Eq. (5), that the sign-changing behavior of the term $(y - (1/2))$ for values of y less than and greater than 0.5 results in such behavior in the lower and upper half of the channel. The effect of non-zero β i.e. varying viscosity model) on the velocity distribution is found to result in a similar behavior except with an increase in the local velocity at the lower half of the channel at higher values of ε_K. These results together with Eq. (5) reveal

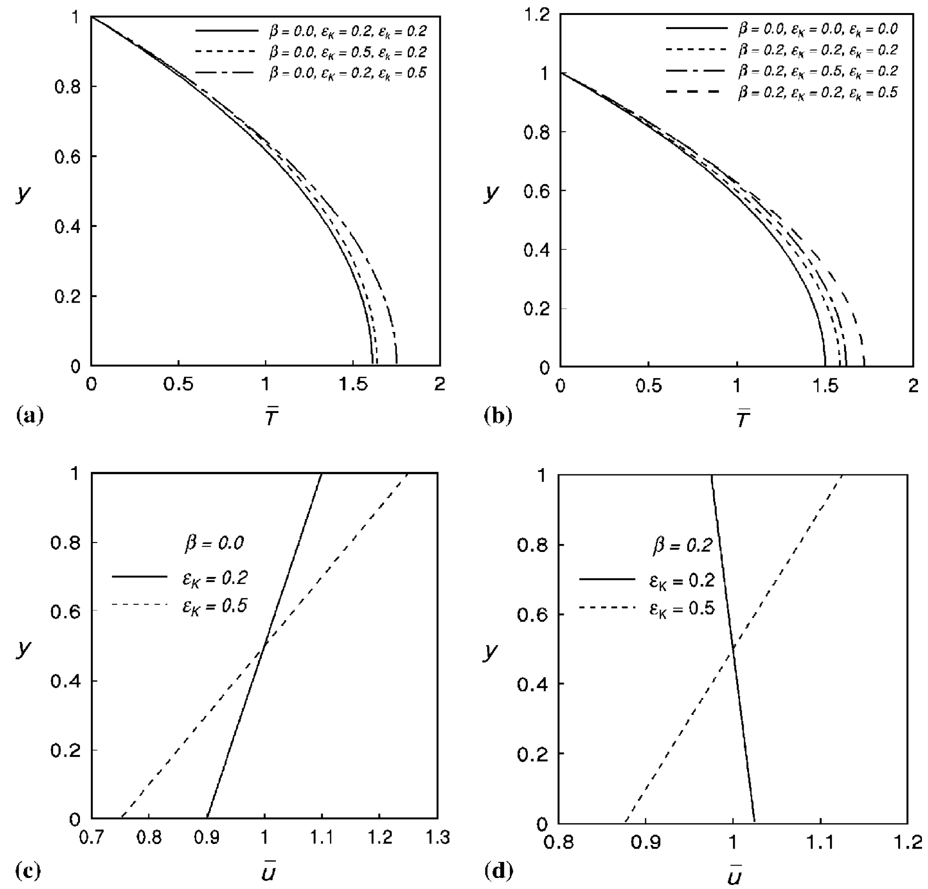

Fig. 3. Temperature and velocity distributions for continuous weak variation case.

that the velocity distribution is independent of thermal conductivity variation ε_k.

The behavior of Nusselt number with ε_k (conductivity variation) is depicted in Fig. 4(a) for different values of ε_K (permeability variation) and $\beta = 0$ (constant viscosity model). The general trend of the Nusselt number is found to increase with increase of both the conductivity and permeability. Also it can be seen from Eq. (21) that, especially when $\beta = 0$, the prime effect of thermal conductivity variation is in the opposite direction to that of permeability variation. The effect of variable viscosity on the Nusselt number is shown in Fig. 4(b) for a fixed ε_K and varying ε_k. With increasing viscosity and conductivity variations, the Nusselt number is found to decrease and increase significantly, which may be explained by the enhancement in lateral mixing, resulting

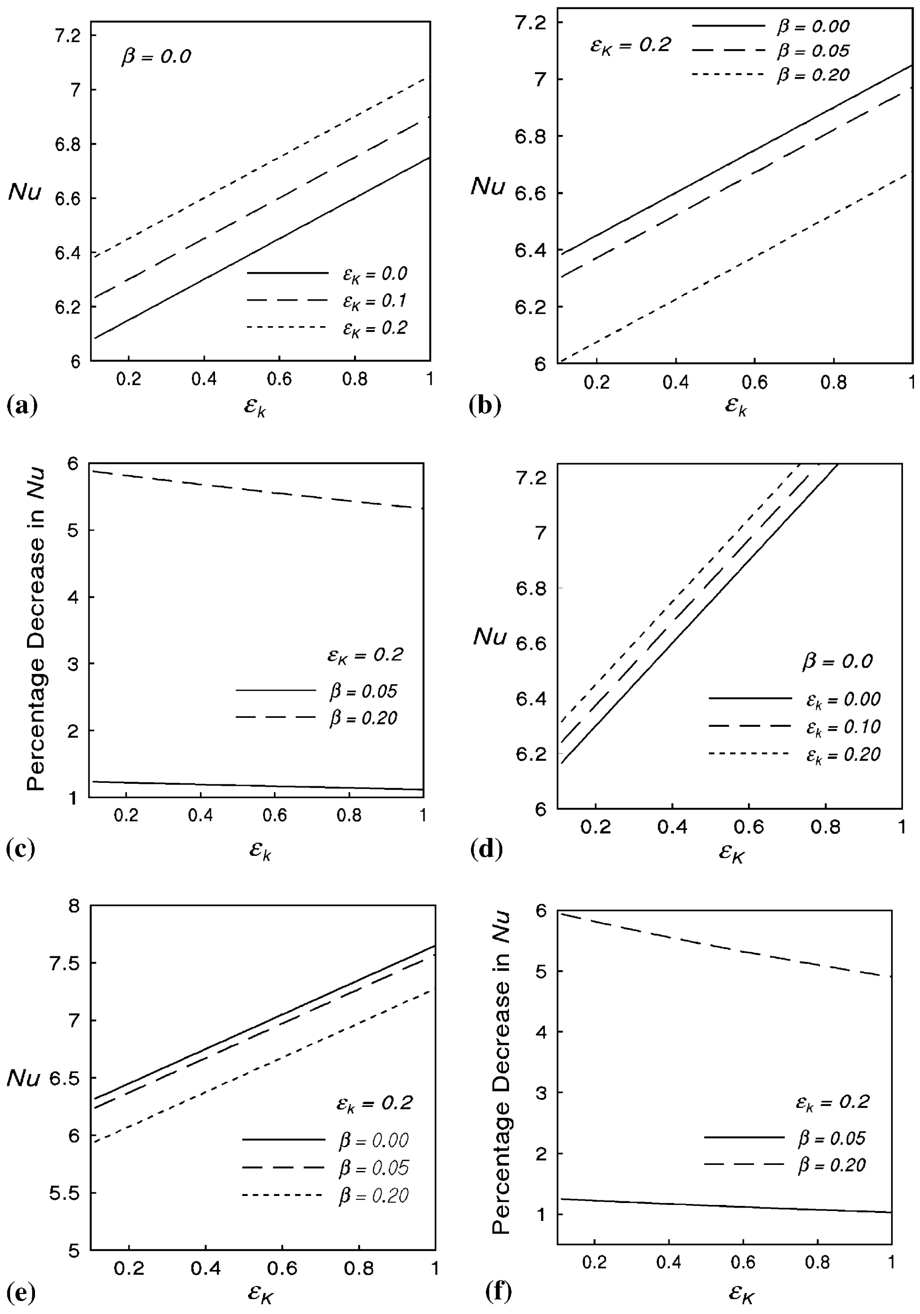

Fig. 4. Variation in Nusselt number with conductivity and permeability for continuous weak variation case.

from the increase of local velocities near the walls for small values of β. The percentage decrease in the Nusselt number compared to that of a constant viscosity model is displayed in Fig. 4(c), for varying conductivity and different values of β. It is found to decrease in small magnitude with conductivity but increase more with β. Fig. 4(d)–(f) display the Nusselt number behavior for a fixed value of conductivity and varying permeability. An increasing trend is noticed in Fig. 4(d) and (e) with ε_K, thus explaining the effect of permeability, whereas the percentage decrease is found to decrease in small magnitude with permeability, seen in Fig. 4(f). Increase in the viscosity variation parameter leads to a decrease the Nusselt number.

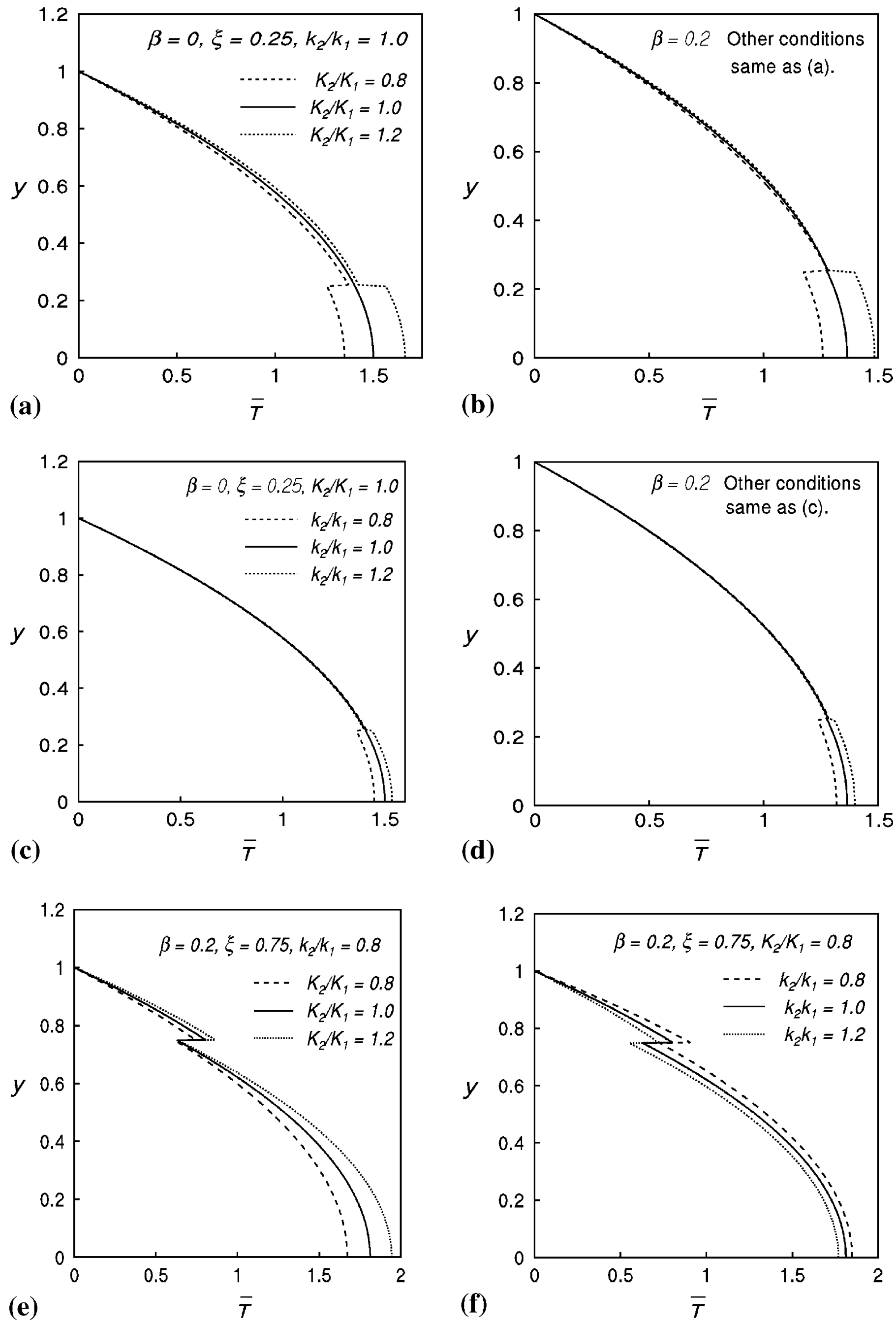

Fig. 5. Temperature distributions for step-wise variation case.

Turning now to the case of step-wise variation, the results are shown in the remaining figures for two different values of ξ viz., 0.25 and 0.75. Fig. 5(a) and (b) show the temperature distributions of both the constant and varying viscosity models across the channel for different combinations of K_2/K_1 and for a fixed value of ξ and k_2/k_1 that is, 0.25 and 1.0 respectively. The temperature distribution is found to increase with the increase of the permeability ratio K_2/K_1 across the channel for both cases. But for the case of varying viscosity model (non-zero β) the temperature is found to be less, especially in Region 1, due to the fact that the enhancement of lateral mixing resulting from the increase in the local velocities near the walls. The variations in the thermal conductivity ratio k_2/k_1 do not seem to affect the temperature distribution significantly as in the above, and are displayed in Fig. 5(c) and (d). The above discussion also holds at higher values of ξ, viz. 0.75 (Fig. 5(e) and (f)), except that the jump (due to the sudden change in permeability) in the temperature distribution is now being observed at $y = 0.75$ instead at $y = 0.25$, as in the above cases.

For $\xi = 0.25$ and different combinations of the thermal conductivity ratio k_2/k_1, the heat transfer rate increases with the permeability ratio K_2/K_1 and are displayed in Fig. 6(a). The effect of viscosity parameter β is found to reduce the Nusselt number, as in the case of the continuous weak variation case, whereas increasing values of thermal conductivity ratio k_2/k_1 lead to increase in the magnitude of the Nusselt number. This decrease in the heat transfer rate with viscosity variation parameter is similar to that of earlier results reported for a homogeneous porous medium [13]. Also for $\xi = 0.75$ and for small values of β, the Nusselt number behaves in a similar manner to that of the case $\xi = 0.25$ with the conductivity ratio as shown in Fig. 6(b). But for large values of β (indication of a strong viscosity variation) and $\xi = 0.75$, the Nusselt number is found to decrease with the permeability ratio, Fig. 6(b). This in turn reveals the importance of the present study. That is, if the heat transfer occurs in flow of a viscous fluid (whose viscosity strongly depends on temperature) through a heterogeneous porous medium, then one should take into account the viscosity variations which otherwise

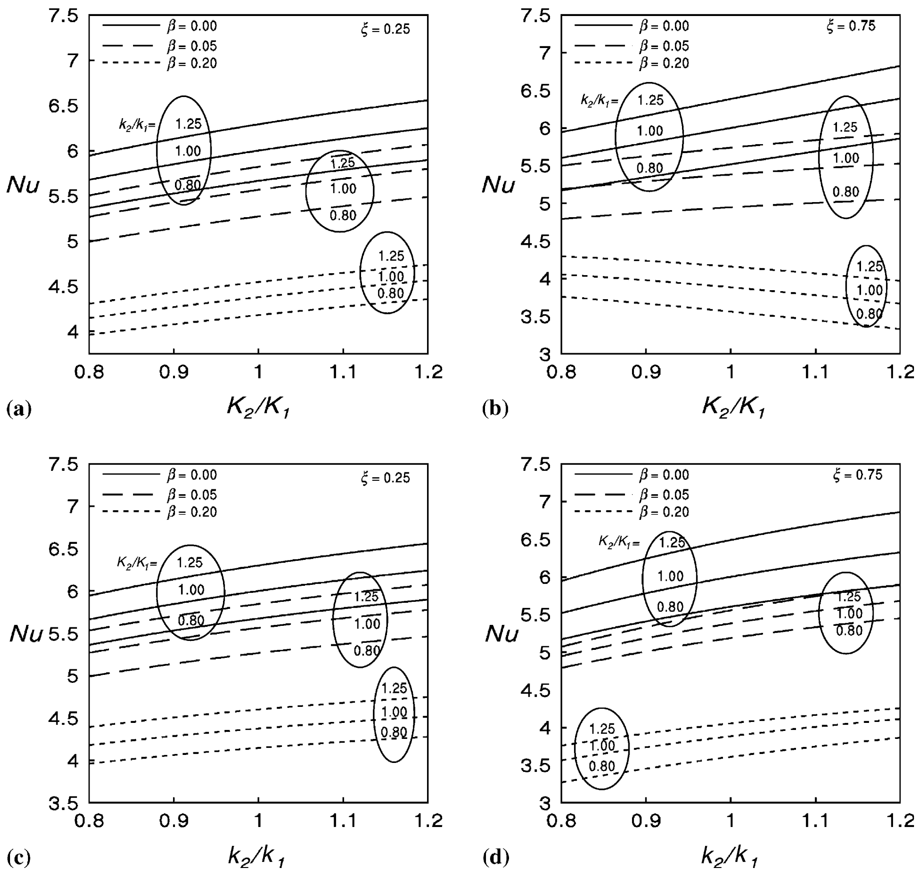

Fig. 6. Variation in Nusselt number with K_2/K_1 and k_2/k_1 for step-wise variation case.

results in an incorrect prediction of the heat transfer rate with the permeability ratio.

Fig. 6(c) and (d) depict the Nusselt number behavior with varying conductivity ratio and for different values of K_2/K_1. Similar trend as in Fig. 6(a) is observed here for the case of $\xi = 0.25$. On examining Fig. 6(d) we infer that even though the general nature of the Nusselt number behavior resembles those in Fig. 6(b) for small values of β, its behavior for large values of β ($= 0.2$) differs with that of Fig. 6(b) resulting in an increase with conductivity ratio.

The percentage decrease in the Nusselt number for two different values of ξ and for variation of the ratios k_2/k_1 and K_2/K_1 are shown in Fig. 7(a)–(d). For $\xi = 0.25$, the percentage decrease is found to be nearly independent of the ratio K_2/K_1 (Fig. 7(a)) but for $\xi = 0.75$, an increase in the percentage decrease in the Nusselt number is noted (Fig. 7(b)) with K_2/K_1. In the case of percentage decrease in Nusselt number with conductivity ratio, it is seen in Fig. 7(c) and (d), that for the same two values of ξ as in Fig. 7(a) and (b), the qualitative behavior is reversed. That is, in Fig. 7(c) an increase in percentage decrease in the Nusselt number is observed with conductivity ratio k_2/k_1, whereas in Fig. 7(d) the percentage decrease is found to be nearly independent of the conductivity ratio.

4. Concluding remarks

An analysis of forced convection flow through a heterogeneous porous medium under isoflux boundary condition is attempted in the presence of viscosity variations. Results discussed in terms of velocity and temperature distributions reveal that the fluid flow and heat transfer characteristics are greatly changed by the variations in the permeability, thermal conductivity and fluid viscosity. The percentage variation in the heat transfer rate, calculated in terms of Nusselt number, for the varying viscosity model is explored by comparing with the earlier constant viscosity model. The effect of viscosity variations taken into consideration here is found to significantly alter the heat transfer rate by reducing it. The prime reason for such reduced heat transfer rate may be attributable to the enhancement of lateral fluid mixing resulting from the increase of local

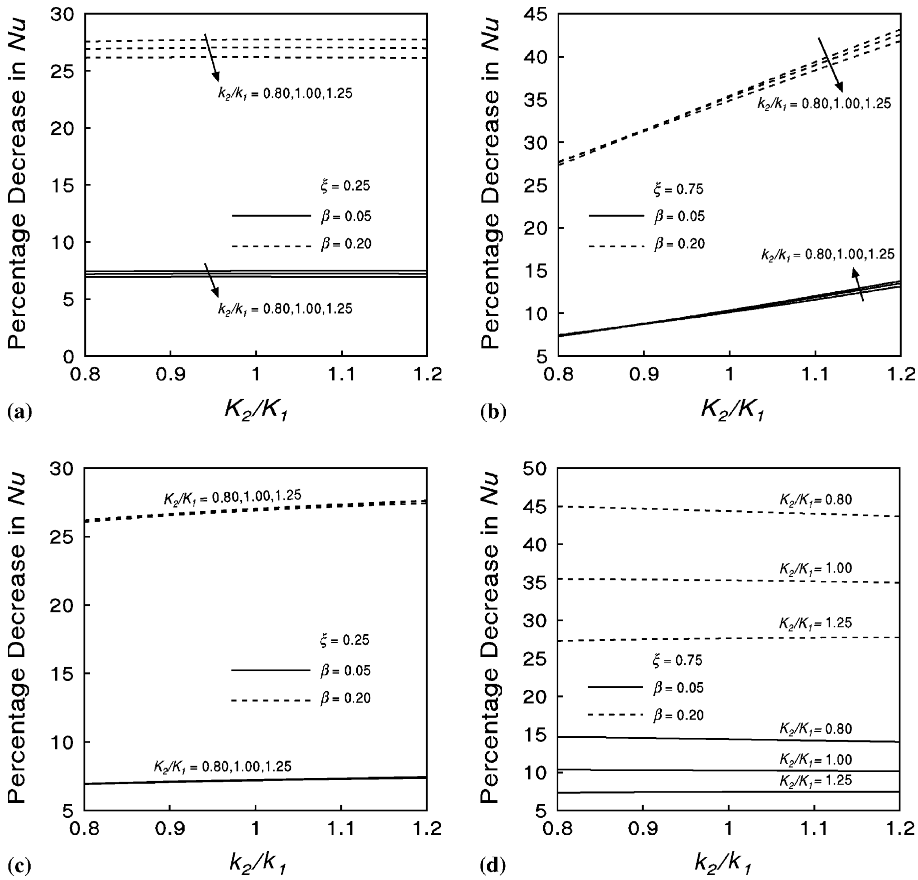

Fig. 7. Percentage decrease in Nusselt number with K_2/K_1 and k_2/k_1 for step-wise variation case.

velocities near the walls, due to change in the fluid viscosity. Further the behavior of the heat transfer rate is found to decrease with the increase of permeability ratio, only for strong viscosity variations, which cannot be captured if one assumes a constant viscosity model.

Appendix A. Justification of Eq. (3)

We know that for most fluids under moderate temperature changes viscosity can be expressed as a linear function of temperature:

$$\mu = \mu_0(1 - \alpha(T' - T'_{\rm w})). \tag{A.1}$$

Introducing the dimensionless temperature $\overline{T}$, Eq. (A.1) is re-written as,

$$\mu = \mu_0[1 - \alpha(T'_{\rm m} - T'_{\rm w})\overline{T}]. \tag{A.2}$$

But from [11], we learn that the $\overline{T}(y)$ distributes almost linearly across the upper half of the channel and therefore we write

$$\overline{T}(y) \cong \gamma(1 - y), \tag{A.3}$$

where γ is a constant.

On using Eq. (A.3), Eq. (A.2) is re-written as

$$\mu = \mu_0[1 - \alpha(T'_{\rm m} - T'_{\rm w})\gamma(1 - y)]. \tag{A.4}$$

Defining

$$\beta = \gamma\alpha(T'_{\rm m} - T'_{\rm w}), \tag{A.5}$$

Eq. (A.4) becomes Eq. (3):

$$\mu = \mu_0[(1 - \beta) + \beta y].$$

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